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Design of Composite Wings Including Uncertainties: A Probabilistic Approach

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A probabilistic method is developed to optimize the design of an idealized composite wing through consideration of the uncertainties in the material properties, fiber-direction angle, and ply thickness. The polynomial chaos expansion method is used to predict the mean, variance, and probability density function of the flutter speed, making use of an efficient Latin hypercube sampling technique. One-dimensional, two-dimensional, and three-dimensional polynomial chaos expansions are introduced into the probabilistic flutter model for different combinations of material, fiber-direction-angle, and ply-thickness uncertainties. The results are compared with Monte Carlo simulation and it is found that the probability density functions obtained using second- and third-order polynomial chaos expansion models compare well but require much less computation. A reliability criterion is defined, indicating the probability of failure due to flutter, and is used to determine successfully the optimal robust design of the composite wing.

Nomenclature

A = inertia matrix
a_{ix} = deterministic coefficients
B = aerodynamic damping matrix
C = aerodynamic stiffness matrix
c_1, c_2 = constants in particle swarm optimization
D = structural damping matrix
E = structural stiffness matrix
g_i = best position found by the entire swarm
I = identity matrix
N = dimension of the search space
n = number of modes or number of Gaussian random variables
O = null matrix
p = order of the Hermite polynomial
p_i = best position found by the ith particle
Q = eigenmatrix of the aeroelastic system
$q_i(t)$ = generalized displacement of the ith mode
V = velocity
v_i = velocity of the ith particle
w = out-of-plane deflection of the wing or inertia factor
x_i = position of the ith particle
β_i = coefficients of the polynomial chaos expansion model
Γ_p = set of multidimensional Hermite polynomials of order p ($\lfloor \xi_{i1}(\theta), \ldots, \xi_{ip}(\theta) \rfloor$)
$\gamma(x, y)$ = assumed mode-shape functions
δ_{ij} = Kronecker delta
λ = eigenvalue
μ = mean flutter speed
$\xi_{i1}(\theta)$ = set of independent standard Gaussian random variables
ρ = density of air
σ = standard deviation
σ_χ = standard deviation
ϕ_{1d}, ϕ_{2d} = uniformly distributed random numbers
χ = random variable
χ_{mean} = mean of standard deviation
$\psi_i(\xi(\theta))$ = set of multidimensional Hermite polynomials

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Introduction

COMPOSITE materials are being used increasingly in aerospace structures. In addition to their attractive strength–weight ratios, they offer the possibility of innovative design concepts, meeting increasing performance demands via the use of their anisotropic properties. There has been much work since the 1980s in the area of aeroelastic tailoring [1–8], in which the composite layup is designed to meet a range of criteria via exploitation of wing bending–torsion coupling: initially, to reduce the divergence problem for forward-swept wing designs [1], but also for weight reduction [2], drag reduction [3], gust response [7,8], optimum flutter characteristics [4], and maintaining static strength, buckling constraints, and optimum flutter and divergence characteristics with a minimum-weight design [5,6].

Although it is possible to model composite wings to a high accuracy, in the real world, uncertainties such as material nonhomogeneity, manufacturing tolerance of the local fiber-direction angle, and thickness variation exist. The most straightforward technique to explore the effect of such uncertainties is Monte Carlo simulation; however, in a wide number of applications, the amount of computation required to give meaningful results is excessive. Consequently, there is a growing need to produce reliable and robust aeroelastic models that incorporate these uncertainties and can predict the effect of uncertainty in a range of parameters.

Uncertainties [9] can be handled using several theories, such as probability theory [10], fuzzy theory [11], evidence theory (also known as Dempster–Shafer theory) [12,13], Bayesian theory, and convex-model theory [14,15]. Information-gap-decision theory under severe uncertainty has also been devised [16]. The common issue among these theories is how to determine the degree to which uncertain events are likely to occur, and there are distinct differences between the various approach theories as to how this is achieved.

Some work has been undertaken on the influence of uncertainties on aeroelastic behavior [17–19]. Various sources of uncertainties can potentially complicate aircraft design, testing, and certification methods, and some work on quantification of various aeroelastic problems such as flutter flight testing, prediction of limit cycle oscillations (LCO), and design optimization with aeroelastic constraints was studied [17]. The LCO of a rigid pitch-plunge airfoil incorporating uncertainties in the cubic coefficient of the torsional spring and also in the initial pitch angle of the airfoil was also investigated [18]. The aeroelastic design optimization with respect to uncertainties in material and structural properties was explored to increase the critical airspeed above that of the baseline wing structure using mass balancing [19]. The flutter boundary and LCO behavior of a metallic wing was studied [20] via the incorporation of stiffness uncertainties; the Karhunen–Loeve (KL) expansion was adopted to define stiffness uncertainties along the span of the wing and perturbation theory was applied to find the response variability. A method for the analysis and design of an aeroelastic system subject to the parametric uncertainties via a hard-inequality-constraints requirement has also been proposed [21]. Uncertainty models were defined given by the norm-bounded perturbation from the nominal value (hypersphere or hyperrectangular), and if hard constraints were not satisfied on the surface, the deformation of the hypersphere via either expansion or contraction was proposed as a solution.

Developments of probabilistic models are possible via direct use of the stochastic expansion. Stochastic expansion can be through either the KL [22] or polynomial chaos expansion (PCE). In the KL expansion, truncated KL series are used to represent the random field and can be implemented in the finite element model, and either perturbation theory or a Neumann expansion can be applied to determine the response variability. The KL expansion requires the covariance function of the process to be expanded in which a priori knowledge of the eigenfunctions is required.

PCE is a method that has been used to explore the variability of response in control [23,24], computational fluid dynamics [25,26], and buckling problems [27]. It is implemented in a similar way to the KL expansion, but does not require expansion of the covariance functions and is simple to implement when determining the response model. The use of PCE for the stability and control of nonlinear problems has been found to be an efficient method even when other techniques such as Lyapunov's method have failed [23]. The potential of PCE is tremendous because of its simplicity, versatility, and computational efficiency within the framework of probability theory.

Once quantification of the response variability of the composite-wing structure has been predicted, it is also important to determine its reliability. The concept of reliability is used here as the measure of safety of the structure in terms of the probability of survival (i.e., avoiding flutter). Reliability is often assessed by analyzing the limit-state function [28], in which the reliability index (the distance from the mean of the limit-state function to the most probable point of failure) is computed to gauge its performance.

Different methods such as the first-order reliability method (FORM) [28], second-order reliability method (SORM), and the Hasofer–Lind method (HL method) have been used to predict the structural reliability. In the FORM approach, a first-order Taylor series expansion of the limit-state function is used, whereas in the HL method, this search is expanded from the mean of the limit-state function to the most probable failure point. If the coefficient of variation of the uncertain parameter is large or the response is nonlinear, FORM predicts a poor reliability and the HL method does not guarantee convergence [28]. In the SORM approach, the limit-state function is expanded up to a second-order Taylor series; Breitung's [29] and Tvedt's [30] methods are often applied, however, these methods do not work when the limit-state function has negative curvature.

Work on the reliability of composite structures has been applied to nonaeroelastic applications and the perturbation method and Neumann expansions incorporated with finite element analysis, MCS, FORM, and HL methods have been are used with some success [31–33].

In this work, the PCE method is used to develop a probabilistic aeroelastic model of an idealized rectangular composite wing modeled using a Rayleigh–Ritz assumed-modes-based structural model and unsteady aerodynamics. The mean, variance, and PDF of the flutter speed are determined based upon a PCE model for which the coefficients are determined using the Latin hypercube sampling technique. Different orders of problem are investigated relating to uncertainty in different combinations of material, fiber-direction angle, and ply thickness. The predictions are compared with Monte Carlo simulations. Finally, the optimal robust composite-layup design of the composite wing is determined using an approach based upon the predicted PDF and particle swarm optimization.

Deterministic Aeroelastic Modeling

The composite lifting surface is idealized as a rectangular plate in a uniform incompressible flow. The out-of-plane deflection in a generalized coordinate system is expressed as

$$w = \sum_{1}^{n} \gamma(x, y) q_i(t) \tag{1}$$

where w is the out-of-plane deflection and $q_i(t)$ is the generalized displacement of the ith mode represented by $\gamma(x, y)$. The Rayleigh–Ritz assumed-modes method [34] is coupled with a simplified but representative unsteady aerodynamic model [22] to give the equation of motions formulated in the classical aeroelastic form:

$$(A)\ddot{q} + (\rho VB + D)\dot{q} + (\rho V^2 C + E)\underline{q} = \underline{0} \tag{2}$$

In first-order matrix form [22], Eq. (2) becomes

$$\begin{pmatrix} \dot{\underline{q}} \\ \ddot{\underline{q}} \end{pmatrix} - \begin{pmatrix} 0 & I \\ -A^{-1}(\rho V^2 C + E) & -A^{-1}(\rho VB + D) \end{pmatrix} \begin{pmatrix} \underline{q} \\ \dot{\underline{q}} \end{pmatrix} = \dot{\underline{x}} - Q\underline{x} = \underline{0} \tag{3}$$

and the eigenvalues of matrix Q lead to the system natural frequencies and damping ratios at a given flight condition. The flutter speed at a given altitude is determined by increasing the air speed or Mach number until one of the damping values becomes negative. In some cases, divergence can arise before flutter, and this is characterized by positive real eigenvalues occurring. This deterministic model is the basis of a conventional flutter analysis but gives no information about the probabilistic behavior of the system.

The baseline deterministic case considered here is a rectangular composite-plate wing with an aspect ratio of 4 and fiber-angle layup of $(-45, -45, 0)_s$. A total of nine modes are assumed for the composite-wing model. The frequency and damping-ratio trends of the first three modes are shown in Fig. 1, and it can be seen that a classical flutter mechanism results from the coupling between two modes.

Stochastic Modeling

Wiener [35] introduced a mathematical model of Brownian motion using a multiple stochastic integral with homogeneous chaos.

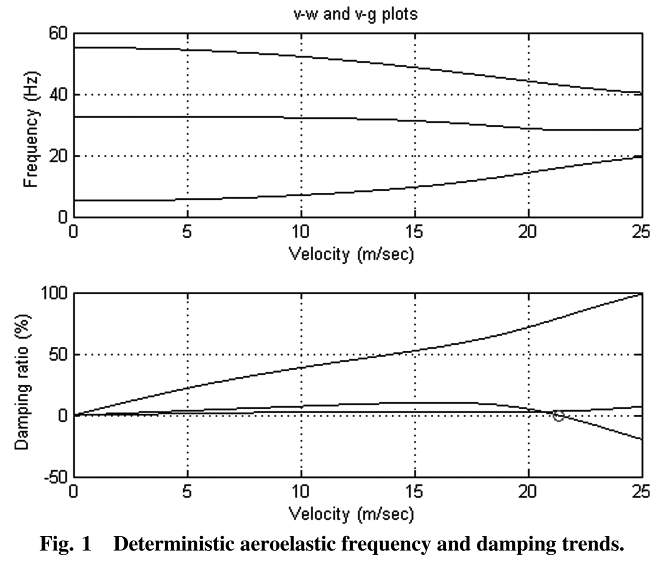

Fig. 1 Deterministic aeroelastic frequency and damping trends.

Subsequently, Ito [36] modified Weiner's [35] work and showed that any stochastic processes can be described as a Weiner process. Irregularities due to parameter variations can be described mathematically as a PCE expansion [37].

Ghanem and Spanos [37] introduced a simple definition of the PCE as a convergent series of the form

$$u(\theta) = a_0\Gamma_0 + \sum_{1}^{\infty} a_{i1}\Gamma_1(\xi_{i1}(\theta)) + \sum_{i1=1}^{\infty}\sum_{i2=1}^{i1} a_{i1i2}\Gamma_2[\xi_{i1}(\theta), \xi_{i2}(\theta)] + \sum_{i1=1}^{\infty}\sum_{i2=1}^{i1}\sum_{i3=1}^{i2} a_{i1i2i3}\Gamma_3[\xi_{i1}(\theta), \xi_{i2}(\theta), \xi_{i3}(\theta)] + \cdots \tag{4}$$

where $\{\xi_{i1}(\theta)\}_1^\infty$ is a set of independent standard Gaussian random variables, $\Gamma_p\lfloor\xi_{i1}(\theta), \ldots, \xi_{ip}(\theta)\rfloor$ is a set of multidimensional Hermite polynomials of order p, $a_{i1}, \ldots, a_{ip}$ are deterministic coefficients, and θ is the random character of the quantities involved. The general expression for a multidimensional Hermite polynomial is given by

$$\Gamma_p[\xi_{i1}(\theta), \ldots, \xi_{ip}(\theta)] = (-1)^n \frac{\partial^n e^{-\frac{1}{2}\xi^T\xi}}{\partial\xi_{i1}(\theta), \ldots, \partial\xi_{ip}(\theta)} \tag{5}$$

where vector ξ consists of n Gaussian random variables.

Simplifying Eq. (4) leads to

$$u(\theta) = \sum_{0}^{p} \beta_i\psi_i(\xi(\theta)) \tag{6}$$

where β_i and $\psi_i(\xi(\theta))$ are identical to $a_{i1}, \ldots, a_{ip}$ and $\Gamma_p\lfloor\xi_{i1}(\theta), \ldots, \xi_{ip}(\theta)\rfloor$, respectively.

Consider the one-dimensional polynomial chaos model; we can expand the random response u using orthogonal polynomials in ξ, which have a known probability distribution (for example, unit normal). If u is a function of random variable χ, for which the mean is χ_{mean} and variance σ_χ^2, then ξ is the normalized variable:

$$\xi = \frac{\chi - \chi_{\text{mean}}}{\sigma_\chi} \tag{7}$$

Hence, the response in one dimension can be expressed as

$$u = \beta_0 + \beta_1\xi + \beta_2(\xi^2 - 1) + \beta_3(\xi^3 - 3\xi) + \beta_4(\xi^4 - 6\xi^2 + 3) + \cdots \tag{8}$$

where the orthogonal polynomials and $\xi(\theta)$ satisfy the following conditions:

$$\psi_0 = 1, \quad \langle\psi_i\rangle = 0, \quad \langle\psi_i\psi_j\rangle = \langle\psi_i^2\rangle\delta_{ij}, \quad \forall\ i, j$$
$$\langle\xi^0\rangle = 1, \quad \langle\xi^k\rangle = 0, \quad \forall\ k\,\text{odd} \quad \text{and} \quad \langle\xi^k\rangle = (k-1)\xi^{k-2} \tag{9}$$

with $\langle.\rangle$ indicating the expected value operation. The β_i terms are unknown coefficients that have to be calculated using computed test data sets. For the case that we are considering here, the u parameter is the flutter speed and the ξ variable might be the value of Young's modulus or the orientation of the composite layers.

If we have one uncertain variable, then it is referred to as a 1-D polynomial chaos model. The order of the model is inferred from the power of ξ. In a 2-D polynomial chaos model, there are two uncertain parameters. Using Eq. (4), the expanded form for a 2-D polynomial chaos model in which θ_1 and θ_2 are uncertain parameters can be written as

$$u_{\text{2nd}} = \beta_0 + \beta_1\xi_1 + \beta_2\xi_2 + \beta_3(\xi_1^2 - 1) + \beta_4\xi_1\xi_2 + \beta_5(\xi_2^2 - 1) \tag{10}$$

Similarly, the 2-D expansion of a third-order PCE model can be written as

$$u_{\text{3rd}} = \beta_0 + \beta_1\xi_1 + \beta_2\xi_2 + \beta_3(\xi_1^2 - 1) + \beta_4\xi_1\xi_2 + \beta_5(\xi_2^2 - 1) + \beta_6(\xi_1^3 - 3\xi_1) + \beta_7(\xi_1^2\xi_2 - \xi_2) + \beta_8(\xi_2^2\xi_1 - \xi_1) + \beta_9(\xi_2^3 - 3\xi_2) \tag{11}$$

Determination of β_i Coefficients for Probabilistic Aeroelastic Modeling

A regression model is fitted based on the computed data to determine the unknown β coefficients described, for example, in Eq. (8) relating the flutter speed calculated using the first-order aeroelastic eigenvalue problem to Young's modulus. An efficient Latin hypercube sampling technique [38] is applied to ensure that the examples cover all portions of the input variable range and produce relatively small variance of the response and that only a relatively few cases need to be considered.

After determining the regression-model coefficients, it is computationally efficient to use this model to emulate many different cases covering the variation of the uncertainty. A PDF of the resulting variation (say, of the flutter speed) can then be determined. Note that there is no restriction on the resulting distribution. It is feasible to curve-fit the PDF to determine the form and coefficients of the distribution, and these can be used to estimate the cumulative distribution function and to decide when enough simulations have been performed.

Examples

To demonstrate the proposed concept, a range of 1-D, 2-D, and 3-D polynomial chaos problems are investigated using the flutter speed of the baseline composite wing as an example. The uncertainties in these examples are introduced via use of the coefficient of variation, which is the measure of dispersion of the data and is defined as the ratio of the standard deviation to the mean of the random variable. The material properties used for each case are defined in Table 1.

Example 1: 1-D Polynomial Chaos

The longitudinal Young's modulus with a coefficient of variation of 0.2 is assumed as the random variable. A total of 10 samples are taken to determine the β_i coefficients and first-, second-, and third-order PCE flutter models are formulated. The mean and variance are calculated using Eq. (13). A total of 1288 Monte Carlo simulations are conducted and the statistics are tabulated in Table 1 in which it can be seen that there is an excellent agreement.

A total of 100,000 emulations are performed to produce the PDF plots of the first-, second-, and third-order PCE flutter models. The computational time required to simulate these PCE models is a matter of seconds. Comparisons of the PDF plots for MCS and first-, second-, and third-order PCE models are shown in Fig. 2. The first-order PCE model shows a poor agreement as it ignores nonlinear terms for response. There is little discrepancy between the second- and third-order PCE models and the MCS model. The first-order PCE model will not be presented in the subsequent examples.

Example 2: 2-D Polynomial Chaos

Here, the fiber-direction angles (with each having a coefficient of variation of 0.01) are treated as random variables. The composite-wing panel layup is $(\theta_1 = -33.75\ \text{deg},\ \theta_2 = 28.125\ \text{deg},\ 90\ \text{deg})_s$, with θ_1 and θ_2 treated as the random variables, and a coefficient of variation of 0.015 is used. This value is quite high: for instance, the −33.75 deg local fiber-direction variation is between −35.269 and −32.231 deg. To build the second- and third-order PCE models, a

Table 1 Material properties used in the examples

Example	E_1, GPa	E_2, GPa	G_{12}, GPa	υ_{12}	Density, kg/m^3
1, 4	98	5.7	5.6	0.28	1520
2, 3	140	10	5.6	0.31	1290

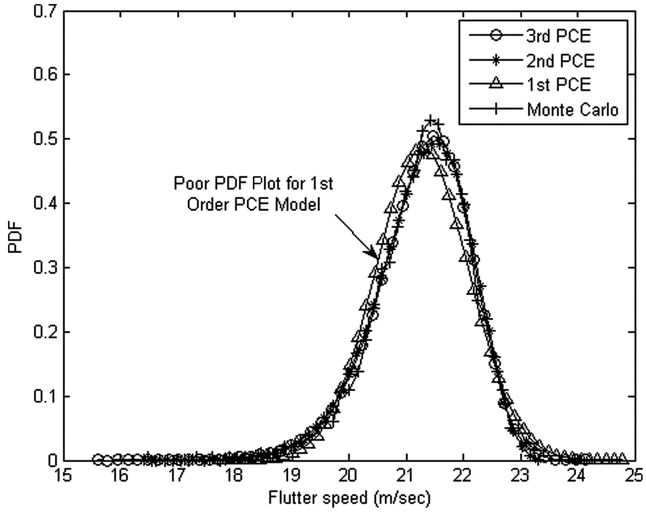

Fig. 2 Example 1: PDF plots of first-, second-, and third-order PCE models and Monte Carlo simulation.

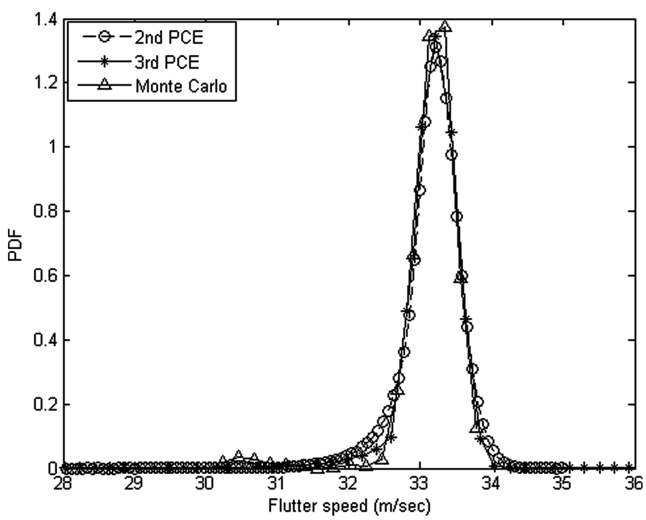

Fig. 3 Example 2: PDF plots of second- and third-order PCE and Monte Carlo simulations.

total of 30 samples are taken. MCS is also performed for results comparison with 2500 simulations, and Fig. 3 and Table 2 show there is good agreement between the two approaches.

Example 3: 3-D Polynomial Chaos

A 3-D polynomial chaos problem is considered with the longitudinal Young's modulus, in-plane shear modulus, and total thickness of the laminate being introduced as variables in the probabilistic flutter model. Assumed coefficients of variation are 0.1 and 0.02 for moduli and laminate thickness, respectively. Results are compared with MCS (900 simulations) in Table 2. Again, the probabilistic model shows a good comparison with the MCS results in Fig. 4. It should be noted from Table 2 that these three examples show that the deterministic flutter speed is often not the same as the mean value of the PDF, due to the possible skewed nature of the resulting flutter-speed distribution.

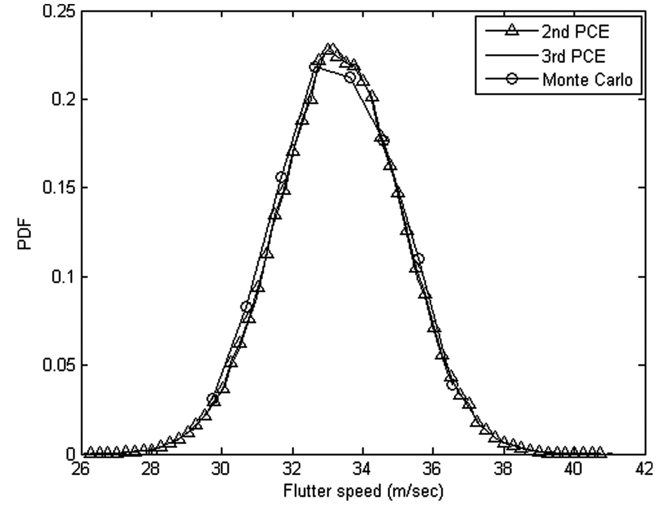

Fig. 4 Example 3: PDF plots of second- and third-order PCE and MCS.

Reliability of the Composite-Plate-Wing Model

The efficient PCE approach to PDF generation can be used for the robust design optimization of composite wings. Here, the concept of reliability is used as a measure of safety of the structure in terms of the probability of survival (i.e., flutter not occurring). Consider the typical PDF plot obtained by applying the PCE method to an optimized deterministic design, as shown in Fig. 5. For some required design flutter speed V_D, the area to the left of this line contains all the uncertain cases that give flutter. However, a robust optimum design is not necessarily the one with the best flutter speed, but the one with the largest PDF area to the right of the design flutter speed. As seen in Fig. 5, the robust PDF is better than the deterministic PDF, even though a significant proportion of the deterministic PDF lies above it. So for a robust optimization, we need to find the design that gives the largest area of PDF above the design flutter speed.

Robust Optimization of the Composite-Plate-Wing Model

Application of Particle Swarm Optimization

Particle swarm optimization (PSO) is a heuristic search method that is based on a simplified social model that is closely tied to swarming theory and intelligence, such that each particle of the swarm has memory and can also communicate with each other [39]. The position and velocity of particles is updated by knowing the previous best values of each particle and overall swarm, such that for the kth iteration,

$$v_{id}(k+1) = wv_{id}(k) + c_1\phi_{1d}(p_{id}(k) - x_{id}(k)) + c_2\phi_{2d}(g_d(k) - x_{id}(k)) \tag{12}$$

$$x_{id}(k+1) = x_{id}(k) + v_{id}(k) \tag{13}$$

where v_i and x_i are the velocity and position of particle i; p_i and g_i are the best positions found by each particle and the entire population; ϕ_{1d} and ϕ_{2d} are independent uniformly distributed random numbers and are generated independently; and w, c_1, and c_2 are the user-defined inertia factor, particle belief factor, and swarm belief factor,

Table 2 Flutter-speed mean and standard deviation for examples 1–3

Example	Deterministic flutter speed m/s	Second-order PCE, μm/s	Third-order PCE, μm/s	Monte Carlo, μm/s	Second-order PCE σ	Third-order PCE σ	Monte Carlo σ
1	21.352	21.2697	21.2700	21.2876	0.8306	0.8441	0.8522
2	33.298	33.1708	33.1420	33.1448	0.3746	0.4701	0.4787
3	33.298	33.2260	33.2277	33.2355	1.7268	1.7487	1.6476

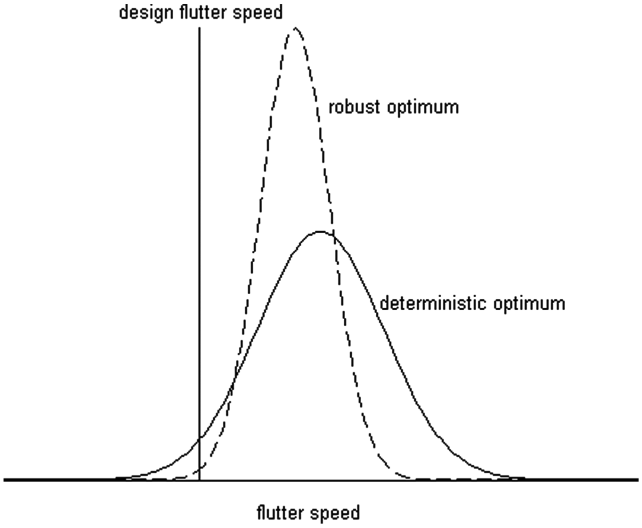

Fig. 5 PDF of flutter speed.

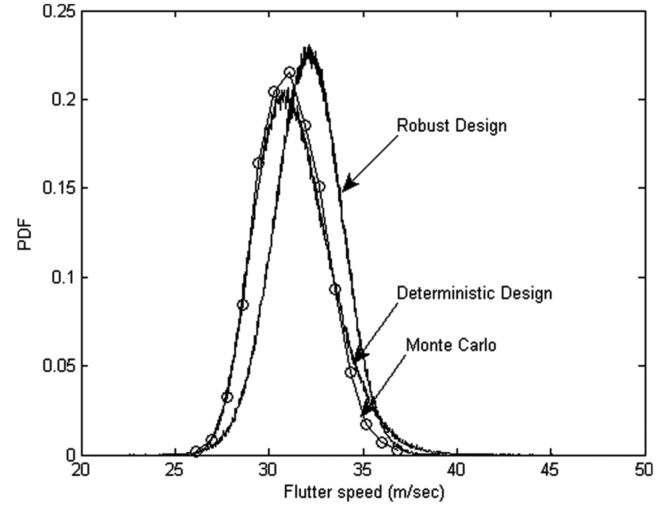

Fig. 6 Example 4: flutter PDFs for deterministic and robust designs.

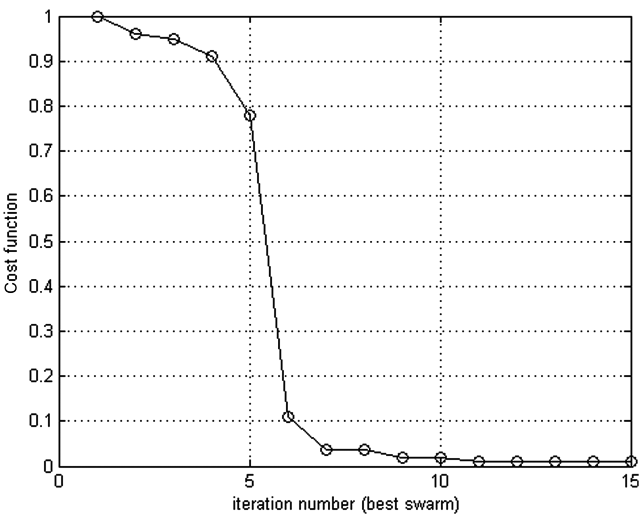

Fig. 7 PSO convergence for the robust optimization case.

respectively. The application of PSO for reliability-based design optimization is shown in the next section.

Example 4: Robust Optimization Including Uncertainty

A composite wing composed of six layers with a layup configuration of $(\theta_1, \theta_2, \theta_3)_s$ is considered as the case to illustrate how the PDFs generated from the PCE approach can be used for robust optimization. Longitudinal Young's modulus E_1, in-plane shear modulus G_{12}, and total thickness of laminate are all treated as uncertain parameters. The coefficient of variation for the moduli is 0.1 and that for the thickness is set as 0.02.

Initially, the wing-layup orientation is optimized using PSO, assuming that there is no uncertainty in the parameters, giving the deterministic maximum flutter speed of 32.9 m/s, as shown in Table 3. The optimum orientations are found to be $[-34.16, 48.5, 21.0]_s$. For every application of PSO, eight particles in the swarm are selected. A second-order probabilistic flutter model is then derived based upon these orientations using 30 different test samples. PDF plots are then generated from this flutter model by taking 100,000 emulations. Comparison with MCS (3375 simulations) is shown in Fig. 6, and once again there is good agreement. The MCS results are performed to confirm the finding that there is a noticeable skewness to the PDF, and Table 3 shows that the mean of the probabilistic model is almost 2 m/s less than the deterministic optimum.

A robust optimization is then performed for two different design flutter speeds: 28 m/s and 32 m/s. For each selected layup orientation, a PCE probabilistic flutter model is generated using the preceding procedure, and then the PDF is calculated using 100,000 emulations. The area of the PDF less than the design flutter speed is then minimized. Figure 7 shows a typical convergence of the optimization process. It can be seen that on the initial runs, the entire PDF distribution is less (i.e., flutter occurs) than the design flutter speed; however, after 10 iterations, virtually all of the PDF distribution is above (flutter-free) the design flutter speed. Figure 6 shows the optimum robust design PDF for the 28 m/s case (for which orientations are found to be $[-33.62, 47.82, 67.9]_s$ and give a flutter speed of 32.24 m/s without uncertainties in the parameters), and it can be clearly seen how most of the area is above that of the deterministic optimum PDF, even though the mean value is less. For the two design cases considered, the robust design approach has roughly a 50% reduction in flutter cases compared with (see Table 4) the deterministic optimum design.

Such an approach could arguably be employed for aircraft certification and not just for composite designs. Currently, a design flutter boundary is specified that must be demonstrated by flight test; however, a flutter-free safety margin (typically 15%) must also be

Table 4 Example 4: probability of failure

Design flutter speed	Deterministic optimum	Robust optimum
28 m/s	0.0292	0.0133
32 m/s	0.6533	0.4845

Table 3 Example 4: flutter speeds for different approaches

	Flutter speed, m/s
Deterministic optimization flutter speed	32.90
Mean of Monte Carlo simulations applied to deterministic optimum	31.15
Mean of PCE applied to deterministic optimum	31.17
Mean of PCE applied to robust optimum	32.10

validated using extrapolation of flight-test results [40]. For a probabilistic approach, the PDFs due to uncertainties (including flight condition) would need to be shown to be above the design flutter speed, rather than simply defining a safety margin. Airworthiness is still a long way from considering such an approach; however, with increasingly efficient structural designs, the influence of uncertainties on flutter boundaries will have to be taken into account some time in the future.

Conclusions

A computationally efficient probabilistic flutter model of a simple composite wing was developed using the polynomial chaos expansion technique and applied successfully to quantify the flutter-speed variation due to the variations in the material, fiber-direction angle, and thickness of the laminate. An excellent agreement of the predicted PDFs was obtained, compared with Monte Carlo simulations.

The reliability of the composite-wing model in terms of the flutter speed was also considered, and a robust design approach incorporating uncertainties was proposed, making use of the efficient PDF generation process. It was shown that a better probabilistic design can be achieved, compared with simply taking the best deterministic design solution.

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